

Supplementary Material

Mechanical stability conditions for 3D and 2D crystals under arbitrary load

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(*Mechanicalstability.nb*)  
Clear["Global*"]  
(*Stiffness_tensor : NiAl_oriented_X = [100]Y = [010]Z = [001]*)  
  
C11all = 190.86794377;  
C22all = 190.86794377;  
C33all = 190.86794377;  
C12all = 142.90799310;  
C13all = 142.90799310;  
C23all = 142.90799310;  
C44all = 121.48571335;  
C55all = 121.48571335;  
C66all = 121.48571335;  
C14all = 0.0;  
C15all = 0.0;  
C16all = 0.0;  
C24all = 0.0;
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C25all = 0.0;
C26all = 0.0;
C34all = 0.0;
C35all = 0.0;
C36all = 0.0;
C45all = 0.0;
C46all = 0.0;
C56all = 0.0(*GPa*);

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(*Stiffness_tensor_in_orthonormal_notation*)

$$ST = \begin{pmatrix} C11all & C12all & C13all & \sqrt{2} * C14all & \sqrt{2} * C15all & \sqrt{2} * C16all \\ C12all & C22all & C23all & \sqrt{2} * C24all & \sqrt{2} * C25all & \sqrt{2} * C26all \\ C13all & C23all & C33all & \sqrt{2} * C34all & \sqrt{2} * C35all & \sqrt{2} * C36all \\ \sqrt{2} * C14all & \sqrt{2} * C24all & \sqrt{2} * C34all & 2 * C44all & 2 * C45all & 2 * C46all \\ \sqrt{2} * C15all & \sqrt{2} * C25all & \sqrt{2} * C35all & 2 * C45all & 2 * C55all & 2 * C56all \\ \sqrt{2} * C16all & \sqrt{2} * C26all & \sqrt{2} * C36all & 2 * C46all & 2 * C56all & 2 * C66all \end{pmatrix};$$

ST//MatrixForm

(*TeXForm[ST]*)

(*Kelvin_moduli*)

EST = Eigenvalues[ST]

$$\begin{pmatrix} 190.868 & 142.908 & 142.908 & 0. & 0. & 0. \\ 142.908 & 190.868 & 142.908 & 0. & 0. & 0. \\ 142.908 & 142.908 & 190.868 & 0. & 0. & 0. \\ 0. & 0. & 0. & 242.971 & 0. & 0. \\ 0. & 0. & 0. & 0. & 242.971 & 0. \\ 0. & 0. & 0. & 0. & 0. & 242.971 \end{pmatrix}$$

{476.684, 242.971, 242.971, 242.971, 47.96, 47.96}

(*Stiffness_tensor : NiAl_oriented_X = [110]Y = [-110]Z = [001]*)

C11all = 288.37368179;

C22all = 288.37368179;

C33all = 190.86794377;

C12all = 45.402255088;

C13all = 142.90799310;

C23all = 142.90799310;

C44all = 121.48571335;

C55all = 121.48571335;

C66all = 23.979975335;

C14all = 0.0;

C15all = 0.0;

C16all = 0.0;

C24all = 0.0;

C25all = 0.0;

C26all = 0.0;

C34all = 0.0;

C35all = 0.0;

C36all = 0.0;

C45all = 0.0;

C46all = 0.0;

C56all = 0.0(*GPa*);

(*Stiffness_tensor_in_orthonormal_notation*)

$$DST = \begin{pmatrix} C11all & C12all & C13all & \sqrt{2} * C14all & \sqrt{2} * C15all & \sqrt{2} * C16all \\ C12all & C22all & C23all & \sqrt{2} * C24all & \sqrt{2} * C25all & \sqrt{2} * C26all \\ C13all & C23all & C33all & \sqrt{2} * C34all & \sqrt{2} * C35all & \sqrt{2} * C36all \\ \sqrt{2} * C14all & \sqrt{2} * C24all & \sqrt{2} * C34all & 2 * C44all & 2 * C45all & 2 * C46all \\ \sqrt{2} * C15all & \sqrt{2} * C25all & \sqrt{2} * C35all & 2 * C45all & 2 * C55all & 2 * C56all \\ \sqrt{2} * C16all & \sqrt{2} * C26all & \sqrt{2} * C36all & 2 * C46all & 2 * C56all & 2 * C66all \end{pmatrix};$$

DST//MatrixForm

(*TeXForm[ST]*)

(*Kelvin_moduli*)

EST = Eigenvalues[ST]

$$\begin{pmatrix} 288.374 & 45.4023 & 142.908 & 0. & 0. & 0. \\ 45.4023 & 288.374 & 142.908 & 0. & 0. & 0. \\ 142.908 & 142.908 & 190.868 & 0. & 0. & 0. \\ 0. & 0. & 0. & 242.971 & 0. & 0. \\ 0. & 0. & 0. & 0. & 242.971 & 0. \\ 0. & 0. & 0. & 0. & 0. & 47.96 \end{pmatrix}$$

{476.684, 242.971, 242.971, 242.971, 47.96, 47.96}

(*Stiffness_tensor : NiAl_oriented_X = [111]Y = [-1 - 12]Z = [1 - 10]*)

C11all = 320.87559442;

C22all = 288.37368177;

C33all = 288.37368177;

C12all = 77.904167747;

C13all = 77.904167747;

C23all = 110.40608041;

C44all = 88.983800684;

C55all = 56.481888013;

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C66all = 56.481888013;
C14all = 0.0;
C15all = 0.0;
C16all = 0.0;
C24all = 0.0;
C25all = 0.0;
C26all = -45.96464569;
C34all = 0.0;
C35all = 0.0;
C36all = 45.964645690;
C45all = 45.964645690;
C46all = 0.0;
C56all = 0.0(*GPa*);

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(*Stiffness_tensor_in_orthonormal_notation*)
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$$\text{DST} = \begin{pmatrix}
C11all & C12all & C13all & \sqrt{2} * C14all & \sqrt{2} * C15all & \sqrt{2} * C16all \\
C12all & C22all & C23all & \sqrt{2} * C24all & \sqrt{2} * C25all & \sqrt{2} * C26all \\
C13all & C23all & C33all & \sqrt{2} * C34all & \sqrt{2} * C35all & \sqrt{2} * C36all \\
\sqrt{2} * C14all & \sqrt{2} * C24all & \sqrt{2} * C34all & 2 * C44all & 2 * C45all & 2 * C46all \\
\sqrt{2} * C15all & \sqrt{2} * C25all & \sqrt{2} * C35all & 2 * C45all & 2 * C55all & 2 * C56all \\
\sqrt{2} * C16all & \sqrt{2} * C26all & \sqrt{2} * C36all & 2 * C46all & 2 * C56all & 2 * C66all
\end{pmatrix};$$

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DST//MatrixForm
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(*TeXForm[ST]*)
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(*Kelvin_moduli*)
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EST = Eigenvalues[ST]
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$$\begin{pmatrix} 320.876 & 77.9042 & 77.9042 & 0. & 0. & 0. \\ 77.9042 & 288.374 & 110.406 & 0. & 0. & -65.0038 \\ 77.9042 & 110.406 & 288.374 & 0. & 0. & 65.0038 \\ 0. & 0. & 0. & 177.968 & 91.9293 & 0. \\ 0. & 0. & 0. & 91.9293 & 112.964 & 0. \\ 0. & -65.0038 & 65.0038 & 0. & 0. & 112.964 \end{pmatrix}$$

{476.684, 242.971, 242.971, 242.971, 47.96, 47.96}

(*Stiffness.tensor : NiAl_oriented_X = [100]Y = [010]Z = [001]_biaxial_strain_C_singular_for_α = 1.15365*)

C11all = 12.351;

C22all = 12.351;

C33all = 54.415;

C12all = 12.375;

C13all = 5.225;

C23all = 5.225;

C44all = 30.863;

C55all = 30.863;

C66all = 42.361;

C14all = 0.0;

C15all = 0.0;

C16all = 0.0;

C24all = 0.0;

C25all = 0.0;

C26all = 0.0;

C34all = 0.0;

C35all = 0.0;

C36all = 0.0;

C45all = 0.0;

C46all = 0.0;

C56all = 0.0(*GPa*);

S11 = -27.046(*GPa*);

S22 = -27.046(*GPa*);

S33 = -20.573(*GPa*);

S12 = 0.0(*PXY*);

S13 = 0.0(*PXZ*);

S23 = 0.0(*PYZELASTICinLAMMPShasCijssuchashere(4 - 23, 5 - 13, 6 - 12)butinthestressesitisswitched4 - 13, 5 - 23, 6 - 12);

(*Stiffness_tensor_in_orthonormal_notation*)

$$ST = \begin{pmatrix} C11all & C12all & C13all & \sqrt{2} * C14all & \sqrt{2} * C15all & \sqrt{2} * C16all \\ C12all & C22all & C23all & \sqrt{2} * C24all & \sqrt{2} * C25all & \sqrt{2} * C26all \\ C13all & C23all & C33all & \sqrt{2} * C34all & \sqrt{2} * C35all & \sqrt{2} * C36all \\ \sqrt{2} * C14all & \sqrt{2} * C24all & \sqrt{2} * C34all & 2 * C44all & 2 * C45all & 2 * C46all \\ \sqrt{2} * C15all & \sqrt{2} * C25all & \sqrt{2} * C35all & 2 * C45all & 2 * C55all & 2 * C56all \\ \sqrt{2} * C16all & \sqrt{2} * C26all & \sqrt{2} * C36all & 2 * C46all & 2 * C56all & 2 * C66all \end{pmatrix};$$

ST//MatrixForm

(*TeXForm[ST]*)

(*Kelvin_moduli*)

EST = Eigenvalues[ST]

(*Det[ST]*)

(*EST[[2]]/EST[[6]];*)

$$H = \begin{pmatrix} S11 & \frac{1}{2}(-S11 - S22) & \frac{1}{2}(-S11 - S33) & -\frac{S23}{\sqrt{2}} & \frac{S13}{\sqrt{2}} & \frac{S12}{\sqrt{2}} \\ \frac{1}{2}(-S11 - S22) & S22 & \frac{1}{2}(-S22 - S33) & \frac{S23}{\sqrt{2}} & -\frac{S13}{\sqrt{2}} & \frac{S12}{\sqrt{2}} \\ \frac{1}{2}(-S11 - S33) & \frac{1}{2}(-S22 - S33) & S33 & \frac{S23}{\sqrt{2}} & \frac{S13}{\sqrt{2}} & -\frac{S12}{\sqrt{2}} \\ -\frac{S23}{\sqrt{2}} & \frac{S23}{\sqrt{2}} & \frac{S23}{\sqrt{2}} & S22 + S33 & S12 & S13 \\ \frac{S13}{\sqrt{2}} & -\frac{S13}{\sqrt{2}} & \frac{S13}{\sqrt{2}} & S12 & S11 + S33 & S23 \\ \frac{S12}{\sqrt{2}} & \frac{S12}{\sqrt{2}} & -\frac{S12}{\sqrt{2}} & S13 & S23 & S11 + S22 \end{pmatrix};$$

$L = ST + H;$

$L//\text{MatrixForm}$

(*Kelvin_moduli*)

$EL = \text{Eigenvalues}[L]$

$$\begin{pmatrix} 12.351 & 12.375 & 5.225 & 0. & 0. & 0. \\ 12.375 & 12.351 & 5.225 & 0. & 0. & 0. \\ 5.225 & 5.225 & 54.415 & 0. & 0. & 0. \\ 0. & 0. & 0. & 61.726 & 0. & 0. \\ 0. & 0. & 0. & 0. & 61.726 & 0. \\ 0. & 0. & 0. & 0. & 0. & 84.722 \end{pmatrix}$$

{84.722, 61.726, 61.726, 56.1524, 22.9886, -0.024}

$$\begin{pmatrix} -14.695 & 39.421 & 29.0345 & 0. & 0. & 0. \\ 39.421 & -14.695 & 29.0345 & 0. & 0. & 0. \\ 29.0345 & 29.0345 & 33.842 & 0. & 0. & 0. \\ 0. & 0. & 0. & 14.107 & 0. & 0. \\ 0. & 0. & 0. & 0. & 14.107 & 0. \\ 0. & 0. & 0. & 0. & 0. & 30.63 \end{pmatrix}$$

{70.5972, -54.116, 30.63, 14.107, 14.107, -12.0292}

(*Stiffness_tensor : NiAl_oriented_X = [100]Y = [010]Z = [001]_biaxial_strain_L_singular_for_α = 1.14739*)

$C11_{all} = 84.27273925;$

C22all = 84.27273925;
C33all = 58.51715980;
C12all = 30.99934025;
C13all = 15.10775280;
C23all = 15.10775280;
C44all = 39.81698766;
C55all = 39.81698766;
C66all = 59.69167350;
C14all = 0.0;
C15all = 0.0;
C16all = 0.0;
C24all = 0.0;
C25all = 0.0;
C26all = 0.0;
C34all = 0.0;
C35all = 0.0;
C36all = 0.0;
C45all = 0.0;
C46all = 0.0;
C56all = 0.0(*GPa*);
PXX = -26.656906690;
PYY = -26.656906690;
PZZ = -20.426956712;
PYZ = 0.0;
PXZ = 0.0;
PXY = 0.0;
S11 = PXX(*GPa*);

$$S22 = PYY(*GPa*);$$

$$S33 = PZZ(*GPa*);$$

$$S12 = PXY(*PXY*);$$

$$S13 = PXZ(*PXZ*);$$

$$S23 = PYZ(*PYZELASTICinLAMMPShasCijssuchashere(4 - 23, 5 - 13, 6 - 12)butinthestresssitisswitched4 -$$

(*Stiffness_tensor_in_orthonormal_notation*)

$$ST = \begin{pmatrix} C11all & C12all & C13all & \sqrt{2} * C14all & \sqrt{2} * C15all & \sqrt{2} * C16all \\ C12all & C22all & C23all & \sqrt{2} * C24all & \sqrt{2} * C25all & \sqrt{2} * C26all \\ C13all & C23all & C33all & \sqrt{2} * C34all & \sqrt{2} * C35all & \sqrt{2} * C36all \\ \sqrt{2} * C14all & \sqrt{2} * C24all & \sqrt{2} * C34all & 2 * C44all & 2 * C45all & 2 * C46all \\ \sqrt{2} * C15all & \sqrt{2} * C25all & \sqrt{2} * C35all & 2 * C45all & 2 * C55all & 2 * C56all \\ \sqrt{2} * C16all & \sqrt{2} * C26all & \sqrt{2} * C36all & 2 * C46all & 2 * C56all & 2 * C66all \end{pmatrix};$$

ST//MatrixForm

(*TeXForm[ST]*)

(*Kelvin_moduli*)

EST = Eigenvalues[ST]

(*Det[ST]*)

(*EST[[2]]/EST[[6]];*)

$$H = \begin{pmatrix} S11 & \frac{1}{2}(-S11 - S22) & \frac{1}{2}(-S11 - S33) & -\frac{S23}{\sqrt{2}} & \frac{S13}{\sqrt{2}} & \frac{S12}{\sqrt{2}} \\ \frac{1}{2}(-S11 - S22) & S22 & \frac{1}{2}(-S22 - S33) & \frac{S23}{\sqrt{2}} & -\frac{S13}{\sqrt{2}} & \frac{S12}{\sqrt{2}} \\ \frac{1}{2}(-S11 - S33) & \frac{1}{2}(-S22 - S33) & S33 & \frac{S23}{\sqrt{2}} & \frac{S13}{\sqrt{2}} & -\frac{S12}{\sqrt{2}} \\ -\frac{S23}{\sqrt{2}} & \frac{S23}{\sqrt{2}} & \frac{S23}{\sqrt{2}} & S22 + S33 & S12 & S13 \\ \frac{S13}{\sqrt{2}} & -\frac{S13}{\sqrt{2}} & \frac{S13}{\sqrt{2}} & S12 & S11 + S33 & S23 \\ \frac{S12}{\sqrt{2}} & \frac{S12}{\sqrt{2}} & -\frac{S12}{\sqrt{2}} & S13 & S23 & S11 + S22 \end{pmatrix};$$

H//MatrixForm

$$L = ST + H;$$

$L//\text{MatrixForm}$

(*Kelvin_moduli*)

$\text{EL} = \text{Eigenvalues}[L]$

$$\begin{pmatrix} 84.2727 & 30.9993 & 15.1078 & 0. & 0. & 0. \\ 30.9993 & 84.2727 & 15.1078 & 0. & 0. & 0. \\ 15.1078 & 15.1078 & 58.5172 & 0. & 0. & 0. \\ 0. & 0. & 0. & 79.634 & 0. & 0. \\ 0. & 0. & 0. & 0. & 79.634 & 0. \\ 0. & 0. & 0. & 0. & 0. & 119.383 \end{pmatrix}$$

{122.416, 119.383, 79.634, 79.634, 53.2734, 51.3732}

$$\begin{pmatrix} -26.6569 & 26.6569 & 23.5419 & 0. & 0. & 0. \\ 26.6569 & -26.6569 & 23.5419 & 0. & 0. & 0. \\ 23.5419 & 23.5419 & -20.427 & 0. & 0. & 0. \\ 0. & 0. & 0. & -47.0839 & 0. & 0. \\ 0. & 0. & 0. & 0. & -47.0839 & 0. \\ 0. & 0. & 0. & 0. & 0. & -53.3138 \end{pmatrix}$$

$$\begin{pmatrix} 57.6158 & 57.6562 & 38.6497 & 0. & 0. & 0. \\ 57.6562 & 57.6158 & 38.6497 & 0. & 0. & 0. \\ 38.6497 & 38.6497 & 38.0902 & 0. & 0. & 0. \\ 0. & 0. & 0. & 32.5501 & 0. & 0. \\ 0. & 0. & 0. & 0. & 32.5501 & 0. \\ 0. & 0. & 0. & 0. & 0. & 66.0695 \end{pmatrix}$$

{143.59, 66.0695, 32.5501, 32.5501, 9.77182, -0.0404144}

(*Stiffness_tensor : Graphene_under_uniaxial_x_strain_C_singular_for_α = 1.2781*)

$\text{C11all} = 13.18872130;$

$$C22_{all} = 179.25865207;$$

$$C33_{all} = 2.2904217754(*C66 \text{ in LAMMPS*});$$

$$C12_{all} = -48.65406717;$$

$$C13_{all} = 0.0; (*C16 \text{ in LAMMPS*})$$

$$C23_{all} = 0.0(*N/m \text{ *}); (*C26 \text{ in LAMMPS*})$$

$$PXX = -38.926434014;$$

$$PYY = -3.3179214930;$$

$$PXY = 0.0;$$

$$S11 = PXX(*N/m \text{ *});$$

$$S22 = PYY(*N/m \text{ *});$$

$$S12 = PXY(*N/m \text{ ELASTIC in LAMMPS has } C_{ij} \text{ such as here } (4 - 23, 5 - 13, 6 - 12) \text{ but in the stress it is switched})$$

$$(*2D_Stiffness_tensor_in_orthonormal_notation*)$$

$$ST = \begin{pmatrix} C11_{all} & C12_{all} & \sqrt{2} * C13_{all} \\ C12_{all} & C22_{all} & \sqrt{2} * C23_{all} \\ \sqrt{2} * C13_{all} & \sqrt{2} * C23_{all} & 2 * C33_{all} \end{pmatrix};$$

$$ST//MatrixForm$$

$$(*Kelvin_moduli*)$$

$$EST = \text{Eigenvalues}[ST]$$

$$H = \begin{pmatrix} S11 & \frac{1}{2}(-S11 - S22) & \frac{S12}{\sqrt{2}} \\ \frac{1}{2}(-S11 - S22) & S22 & \frac{S12}{\sqrt{2}} \\ \frac{S12}{\sqrt{2}} & \frac{S12}{\sqrt{2}} & S11 + S22 \end{pmatrix};$$

$$H//MatrixForm$$

$$L = ST + H;$$

$$L//MatrixForm$$

$$(*Kelvin_moduli*)$$

EL = Eigenvalues[L]

$$\begin{pmatrix} 13.1887 & -48.6541 & 0. \\ -48.6541 & 179.259 & 0. \\ 0. & 0. & 4.58084 \end{pmatrix}$$

{192.463, 4.58084, -0.0157217}

$$\begin{pmatrix} -38.9264 & 21.1222 & 0. \\ 21.1222 & -3.31792 & 0. \\ 0. & 0. & -42.2444 \end{pmatrix}$$

$$\begin{pmatrix} -25.7377 & -27.5319 & 0. \\ -27.5319 & 175.941 & 0. \\ 0. & 0. & -37.6635 \end{pmatrix}$$

{179.632, -37.6635, -29.4286}

(*Stiffness_tensor : Graphene_under_uniaxial_x_strain_L_singular_for_α = 1.2386*)

C11all = 40.5787048978;

C22all = 194.428534180;

C33all = 37.50271882554(*C66 in LAMMPS*);

C12all = -42.9260293507;

C13all = 0.0; (*C16 in LAMMPS*)

C23all = 0.0(*N/m *); (*C26 in LAMMPS*)

PXX = -38.0970124614324;

PYY = -4.21462319136738;

PXY = 0.0;

S11 = PXX(*N/m *);

$$S22 = PYY(*N/m *);$$

$$S12 = PXY(*N/mELASTICinLAMMPShasCijssuchashere(4 - 23, 5 - 13, 6 - 12)butinthestressesitisswitched4$$

$$(*2D_Stiffness_tensor_in_orthonormal_notation*)$$

$$ST = \begin{pmatrix} C11all & C12all & \sqrt{2} * C13all \\ C12all & C22all & \sqrt{2} * C23all \\ \sqrt{2} * C13all & \sqrt{2} * C23all & 2 * C33all \end{pmatrix};$$

$$ST//MatrixForm$$

$$(*Kelvin_moduli*)$$

$$EST = Eigenvalues[ST]$$

$$H = \begin{pmatrix} S11 & \frac{1}{2}(-S11 - S22) & \frac{S12}{\sqrt{2}} \\ \frac{1}{2}(-S11 - S22) & S22 & \frac{S12}{\sqrt{2}} \\ \frac{S12}{\sqrt{2}} & \frac{S12}{\sqrt{2}} & S11 + S22 \end{pmatrix};$$

$$H//MatrixForm$$

$$L = ST + H;$$

$$L//MatrixForm$$

$$(*Kelvin_moduli*)$$

$$EL = Eigenvalues[L]$$

$$\begin{pmatrix} 40.5787 & -42.926 & 0. \\ -42.926 & 194.429 & 0. \\ 0. & 0. & 75.0054 \end{pmatrix}$$

$$\{205.595, 75.0054, 29.4123\}$$

$$\begin{pmatrix} -38.097 & 21.1558 & 0. \\ 21.1558 & -4.21462 & 0. \\ 0. & 0. & -42.3116 \end{pmatrix}$$

$$\begin{pmatrix} 2.48169 & -21.7702 & 0. \\ -21.7702 & 190.214 & 0. \\ 0. & 0. & 32.6938 \end{pmatrix}$$

$$\{192.705, 32.6938, -0.00980608\}$$

(*Stress_as_second_order_tensor*)

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{pmatrix};$$

σ //MatrixForm

(*Stress_in_orthonormal_notation*)

$$\sigma_o = \{\sigma_{11}, \sigma_{22}, \sigma_{33}, \sqrt{2}\sigma_{23}, \sqrt{2}\sigma_{13}, \sqrt{2}\sigma_{12}\};$$

σ_o //MatrixForm

(*Stress_in_Voigt_notation*)

$$\sigma_V = \{\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{23}, \sigma_{13}, \sigma_{12}\};$$

σ_V //MatrixForm

(*Frobenius_norm_of_sigma, sigma_o_and_sigma_V*)

$$(\text{Norm}[\sigma, \text{"Frobenius"}])^2$$

$$(\text{Norm}[\sigma_o, \text{"Frobenius"}])^2$$

$$(\text{Norm}[\sigma_V, \text{"Frobenius"}])^2$$

$$\begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{pmatrix}$$

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sqrt{2}\sigma_{23} \\ \sqrt{2}\sigma_{13} \\ \sqrt{2}\sigma_{12} \end{pmatrix}$$

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{pmatrix}$$

$$\text{Abs}[\sigma_{11}]^2 + 2\text{Abs}[\sigma_{12}]^2 + 2\text{Abs}[\sigma_{13}]^2 + \text{Abs}[\sigma_{22}]^2 + 2\text{Abs}[\sigma_{23}]^2 + \text{Abs}[\sigma_{33}]^2$$

$$\text{Abs}[\sigma_{11}]^2 + 2\text{Abs}[\sigma_{12}]^2 + 2\text{Abs}[\sigma_{13}]^2 + \text{Abs}[\sigma_{22}]^2 + 2\text{Abs}[\sigma_{23}]^2 + \text{Abs}[\sigma_{33}]^2$$

$$\text{Abs}[\sigma_{11}]^2 + \text{Abs}[\sigma_{12}]^2 + \text{Abs}[\sigma_{13}]^2 + \text{Abs}[\sigma_{22}]^2 + \text{Abs}[\sigma_{23}]^2 + \text{Abs}[\sigma_{33}]^2$$